Online bipartite matching with imperfect advice

Lightning talk @ TTIC Workshop on Learning-Augmented Algorithms

Davin Choo, Themis Gouleakis, Chun Kai Ling, Arnab Bhattacharyya

I’m on the job market!

Online bipartite matching

- Offline set $U = \{u_1, \dots, u_n\}$ fixed and known
- Online set $V = \{v_1, \dots, v_n\}$ arrive one by one

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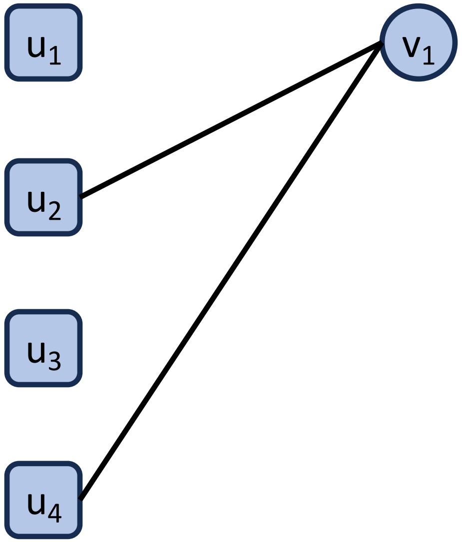

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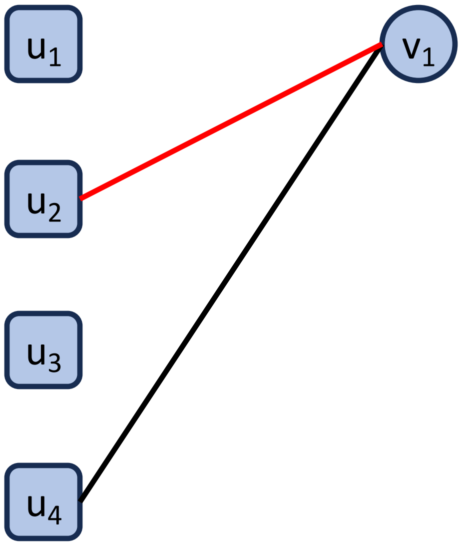

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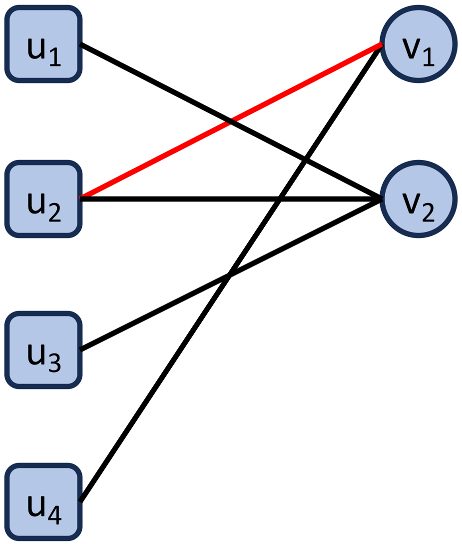

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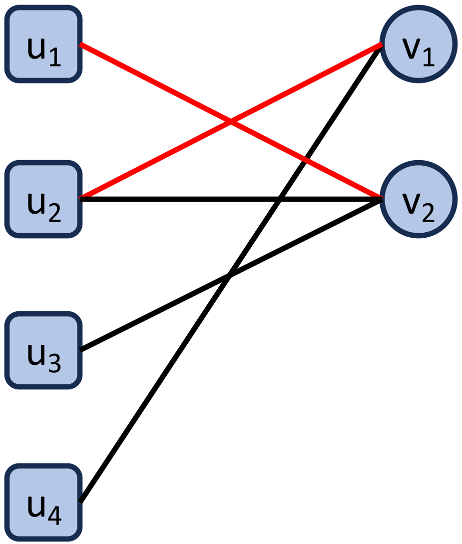

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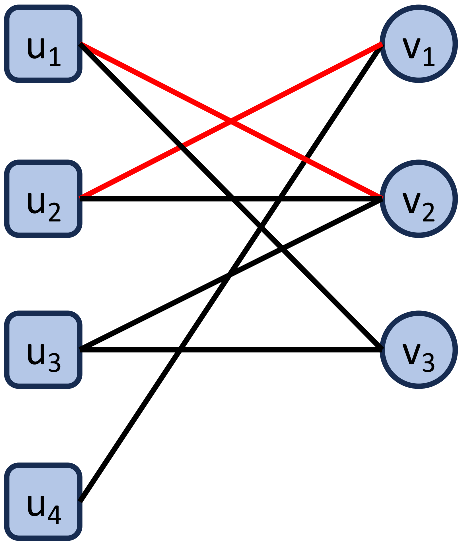

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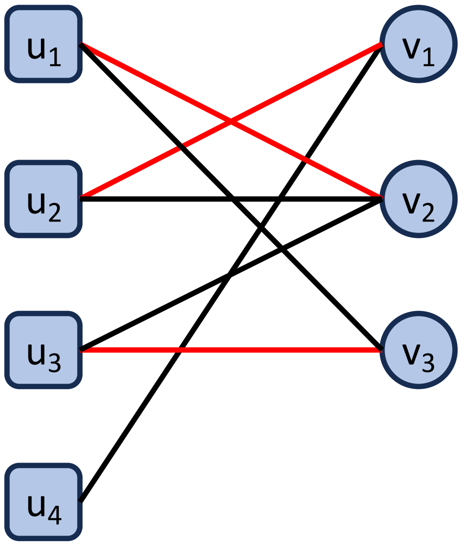

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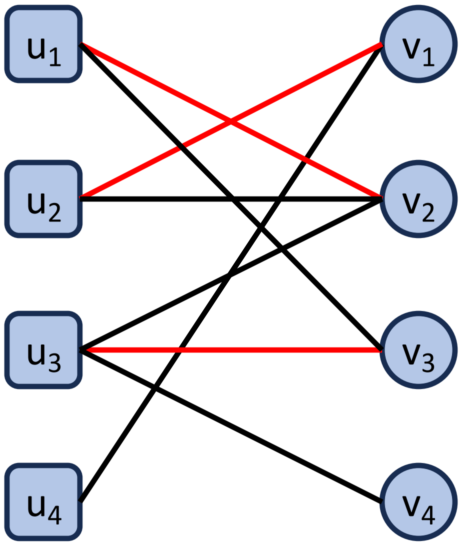

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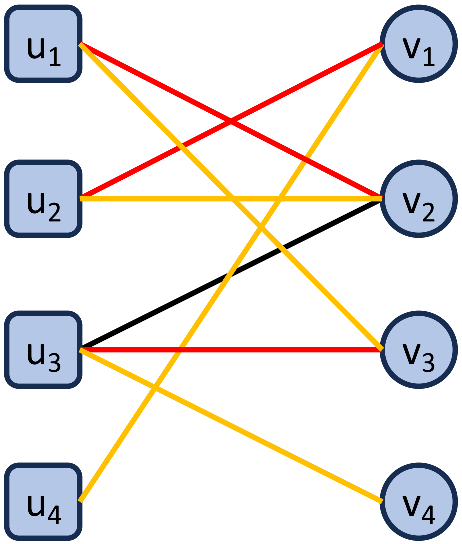
  - We must make an irrevocable decision whether, and how, to match v_i to something in $N(v_i)$
- Final offline graph $G^* = (U \cup V, E)$
  - $E = N(v_1) \cup \cdots \cup N(v_n)$
  - Maximum matching $M^* \subseteq E$ of size $|M^*| = n^* \leq n$



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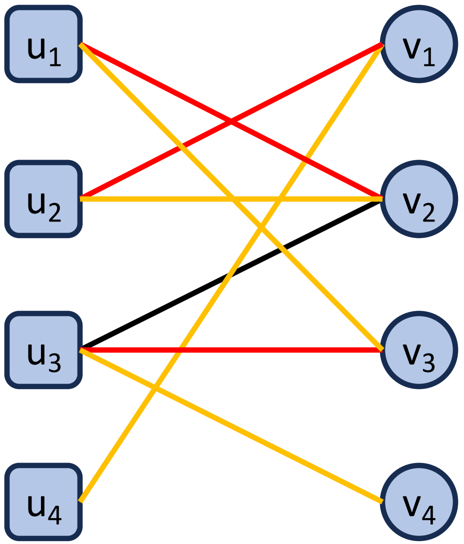

Here, the ratio is $3/4$

Goal of online bipartite matching problem

Produce a matching M such that the resulting competitive ratio $\frac{|M|}{|M^*|}$ is **maximized**

For this talk, let's treat $n^* = n$

What is known?

$$\min_{G} \min_{\substack{V'\text{s arrival}\\ \text{sequence}}} \frac{\text{(Expected) number of matches}}{n^*}$$

	(Expected) Competitive ratio
Deterministic algorithm	$\frac{1}{2}$ ← Greedy
Deterministic hardness	$\frac{1}{2}$
Randomized algorithm	$1-\frac{1}{e}$ [KVV90] ← Ranking
Randomized hardness	$1-\frac{1}{e}+o(1)$ [KVV90]

- The Ranking algorithm [KVV90]
  - Pick a random permutation π over the offline vertices U
  - When vertex v_i arrive with $N(v_i)$, match v_i to the smallest indexed (with respect to π) unmatched neighbor

[KVV90] Richard M. Karp, Umesh V. Vazirani, Vijay V. Vazirani. *An optimal algorithm for on-line bipartite matching*. Symposium on Theory of Computing (STOC), 1990

What if there is additional side information?

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - α-consistent: α-competitive with no advice error
    - β-robust: β-competitive with any advice error

A natural goal is to design an algorithm with $\alpha = 1$ while β being the best possible classically

Research question

- If we have "perfect information" about G^*, can we get n^* matches?
- Also, we know that `Ranking` achieves competitive ratio of $1 - \frac{1}{e}$

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust?

Prior related attempts

- [AGKK20] Prediction on edge weights adjacent to V under an optimal offline matching
  - Random vertex arrivals and weighted edges
  - Require hyper-parameter to quantify confidence in advice, so their consistency/robustness tradeoffs are not directly comparable
- [ACI22] Prediction of vertex degrees $\hat{d}(u_1), \ldots, \hat{d}(u_n)$ of the offline vertices in U
  - Adversarial arrival model
  - Optimal under the Chung-Lu-Vu random graph model [CLV03]
  - Unable to attain 1-consistency in general
- [JM22] Advice is a proposed matching for the first batch of arrived vertices
  - Two-staged arrival model [FNS21], where best possible robustness is ¾
  - For any $R \in [0, ¾]$, they can achieve consistency of $1 - \left(1 - \sqrt{1-R}\right)^2$
- [LYR23] Augment any "expert algorithm" with a pre-trained RL model
  - For any $\rho \in [0,1]$, their method is ρ-competitive to the given "expert algorithm"

[AGKK20] Antonios Antoniadis, Themis Gouleakis, Pieter Kleer, and Pavel Kolev. *Secretary and online matching problems with machine learned advice*. Neural Information Processing Systems (NeurIPS), 2020
[ACI22] Anders Aamand, Justin Chen, and Piotr Indyk. *(Optimal) Online Bipartite Matching with Degree Information*. Neural Information Processing Systems (NeurIPS), 2022
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Our first main result

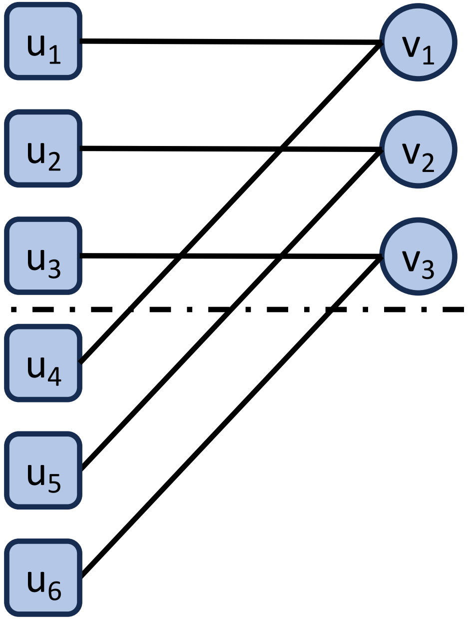
Impossibility result (Informal)

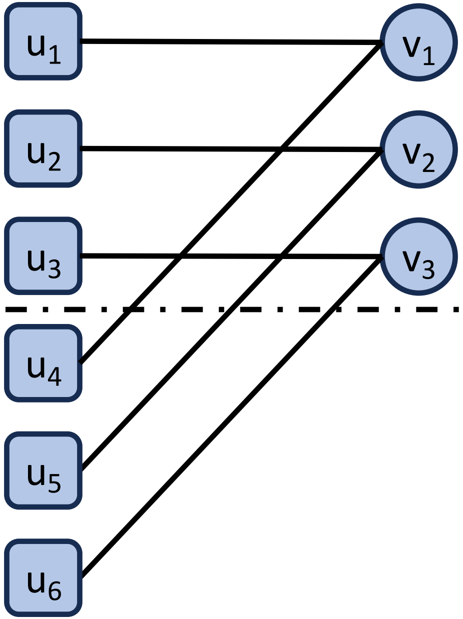
With adversarial vertex arrivals, no algorithm can be both
1-consistent and $> \frac{1}{2}$-robust, regardless of advice

- Extends to $(1 - \mathrm{a})$-consistent and $\left(\frac{1}{2} + \mathrm{a}\right)$-robust, for any $\mathrm{a} \in [0, ½]$.
- Proof sketch (for $\mathrm{a} = 0$ case):
  - Restrict G^* to be one of two possible graphs (next slide)
  - **Any** advice is equivalent to getting 1 bit of information
  - In first $\frac{\mathrm{n}}{2}$ arrivals, no algorithm can distinguish between the two graphs
  - **Any** 1-consistent algorithm must behave as if the advice is perfect initially

Impossibility result (Informal)

With adversarial vertex arrivals, no algorithm can be both 1-consistent and $> \frac{1}{2}$-robust, regardless of advice





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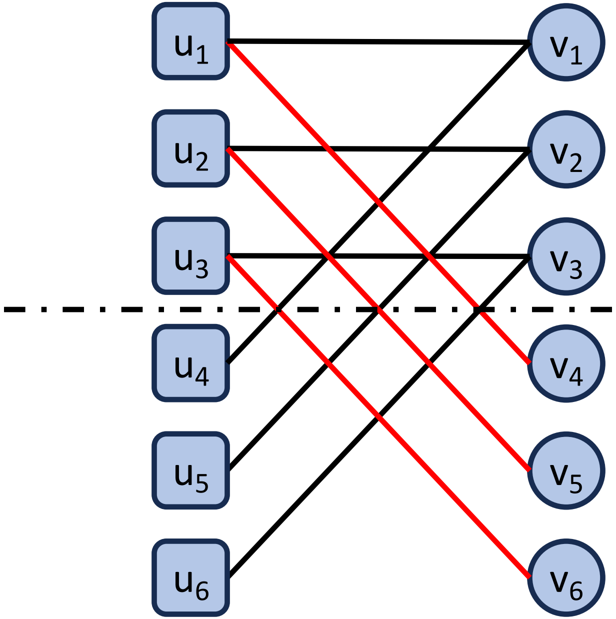

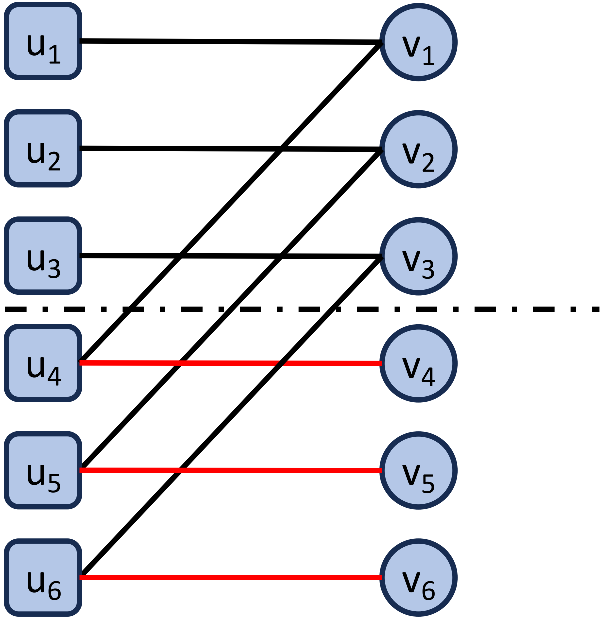

Hierarchy of arrival models [M13]

Harder $\longrightarrow$ Easier

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

Easier models can achieve higher competitive ratios

[M13] Aranyak Mehta. *Online matching and ad allocation*. Foundations and Trends in Theoretical Computer Science, 2013

Hierarchy of arrival models [M13]

Worst case G^*

Each online vertex is drawn from some type distribution $\mathcal{D}: 2^U \to \mathbb{R}$ in an IID fashion

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

Worst case arrival sequence

Arrival sequence is a random permutation

$\mathcal{D}: 2^U \to \mathbb{R}$ unknown

$\mathcal{D}: 2^U \to \mathbb{R}$ known

What is known?

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

	(Expected) Competitive ratio	
	Adversarial arrival	Random order arrival
Deterministic algorithm	$\frac{1}{2}$ Greedy	$1-\frac{1}{e}$ [GM08]
Deterministic hardness	$\frac{1}{2}$	$\frac{3}{4}$
Randomized algorithm	$1-\frac{1}{e}$ [KVV90] Ranking	0.696 [MY11]
Randomized hardness	$1-\frac{1}{e}+o(1)$ [KVV90]	0.823 [MGS12]

[GM08] Gagan Goel and Aranyak Mehta. *Online budgeted matching in random input models with applications to Adwords*. Symposium on Discrete Algorithms (SODA), 2008
[MY11] Mohammad Mahdian and Qiqi Yan. *Online Bipartite Matching with Random Arrivals: An Approach Based on Strongly Factor-Revealing LPs*. Symposium on Theory of Computing (STOC), 2011
[MGS12] Vahideh H Manshadi, Shayan Oveis Gharan, and Amin Saberi. *Online stochastic matching: Online actions based on offline statistics*. Mathematics of Operations Research, 2012

Research question

Can we get an algorithm that is both 1-consistent and $\cancel{\left(1-\frac{1}{e}\right)}\ \beta$-robust?

- Let β denote the “best possible competitive ratio”
- Our first result says: This is not possible for adversarial arrivals!
- What about random order arrivals?

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

Our second main result

Can we get an algorithm that is both 1-consistent and $\cancel{\left(1-\frac{1}{e}\right)}$ β-robust?

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and $\beta \cdot \big(1 - o(1)\big)$, depending on advice quality

- Our method is a meta-algorithm that uses any `Baseline` that achieves β
- So, we are simultaneously 1-consistent and $\beta \cdot \big(1 - o(1)\big)$-robust
- For random arrival model, we know that $0.696 \leq \beta \leq 0.823$

e.g. use `Ranking`

Realized type counts as advice

- Classify online vertex in $G^* = (U \cup V, E)$ based on their types
  - Type of v_i is the set of offline vertices in $N(v_i)$ are adjacent to [BKP20]
- Define integer vector $c^* \in \mathbb{N}^{2^n}$ indexed by all possible types 2^U
  - $c^*(t)$ = Number of times the type $t \in 2^U$ occurs in G^*
- Define $T^* \subseteq 2^U$ as the subset of non-zero counts in c^*
  - Note: $|T^*| \leq n \ll 2^{|U|} = 2^n$

[BKP20] Allan Borodin, Christodoulos Karavasilis, and Denis Pankratov. *An experimental study of algorithms for online bipartite matching*. Journal of Experimental Algorithmics (JEA), 2020

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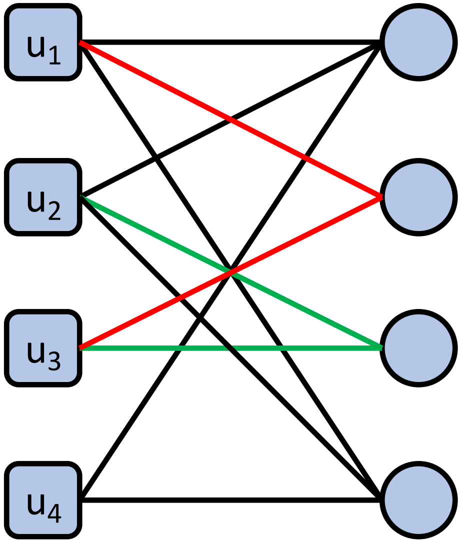
- Define $T^* \subseteq 2^U$ as the subset of non-zero counts in c^*
  - Note: $|T^*| \leq n \ll 2^{|U|} = 2^n$
- Advice is simply an estimate vector $\hat{c}$ which approximates c^*
  - Let $\hat{T}$ be non-zero counts in $\hat{c}$. Similarly, we have $|\hat{T}| \leq n$
  - Can represent $\hat{c}$ using $O(n)$ labels and numbers

Realized type counts as advice



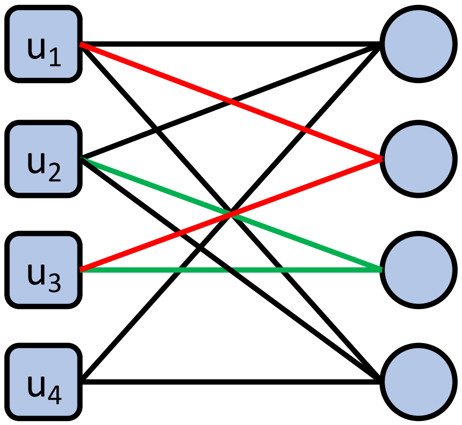
T^*

Type	c^*
$\{u_1, u_2, u_4\}$	2
$\{u_1, u_3\}$	1
$\{u_2, u_3\}$	1
$2^U \setminus T^*$	0

Here, $|T^*| = 3 \ll 2^4 = 16$

The Mimic algorithm

- Algorithm
  - Fix any arbitrary maximum matching $\widehat{M}$ on the graph defined by advice $\hat{c}$
  - Try to mimic edge matches in $\widehat{M}$ while tracking the types of each arrival
  - If unable to mimic, leave arrival unmatched



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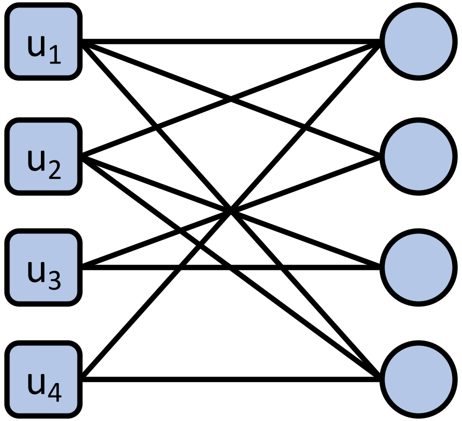

Type	c^*	$\hat{c}$
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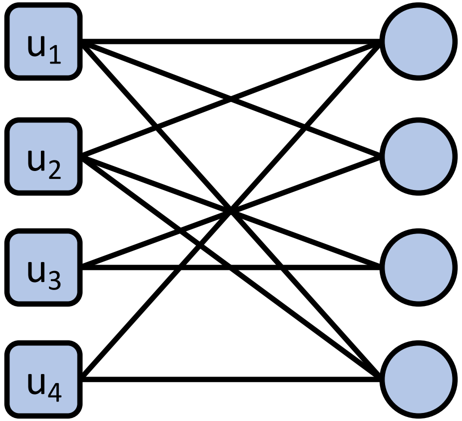

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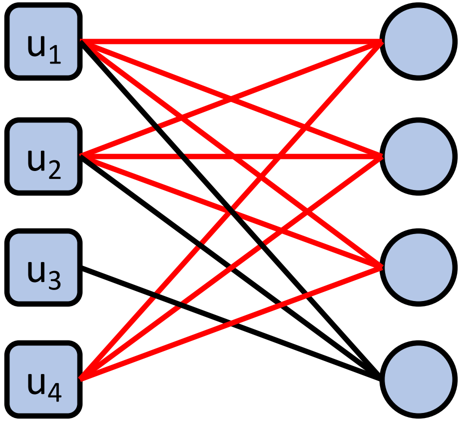

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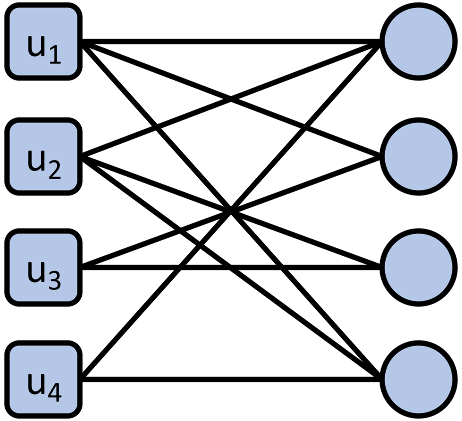

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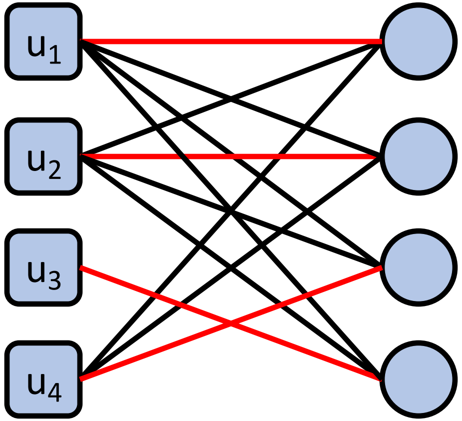

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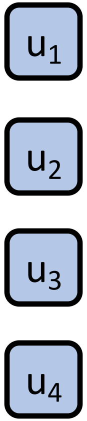

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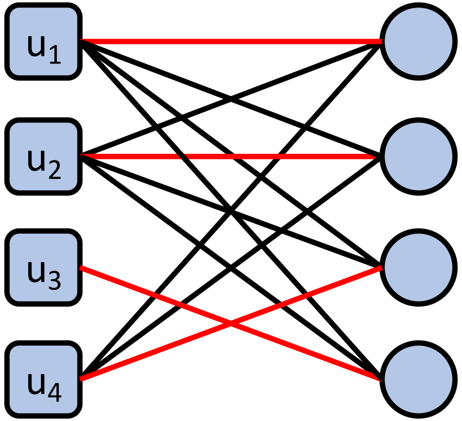

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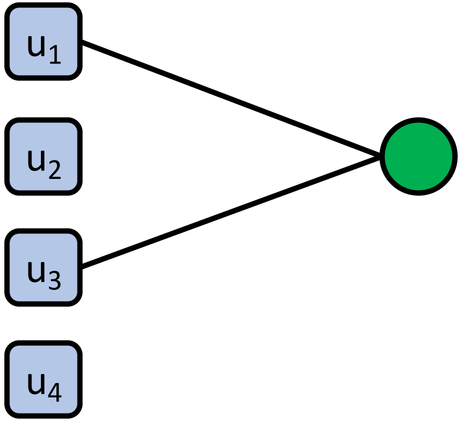

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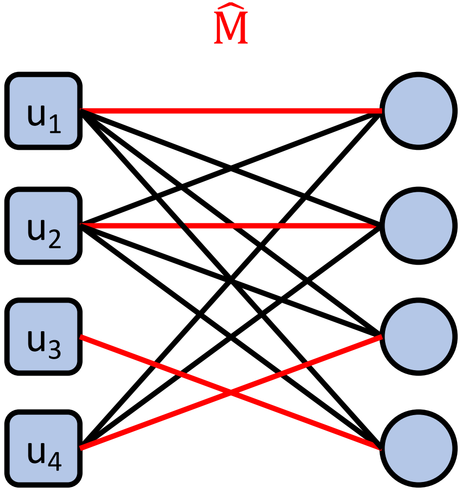

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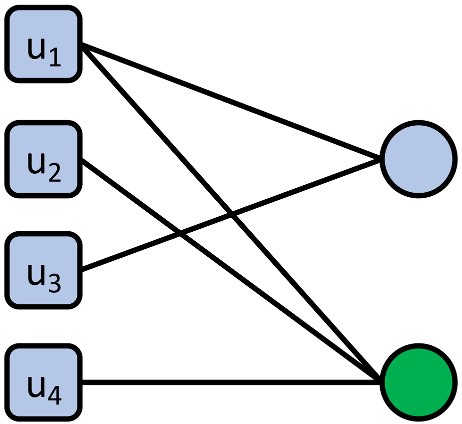

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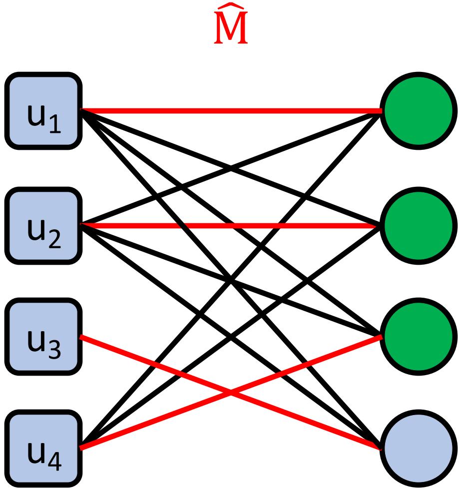

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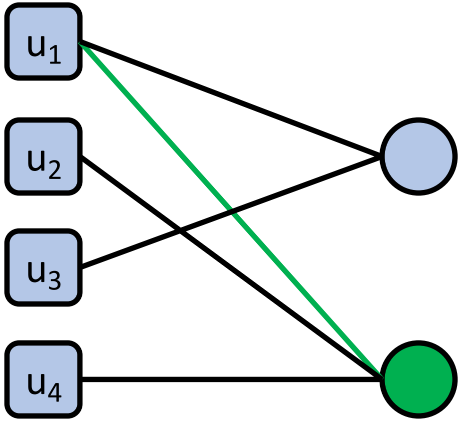
  - Try to mimic edge matches in $\widehat{M}$ while tracking the types of each arrival
  - If unable to mimic, leave arrival unmatched



Type	c^*	$\hat{c}$
$\{u_1, u_2, u_4\}$	2	~~3~~ 2
$\{u_1, u_3\}$	1	0
$\{u_2, u_3\}$	1	0
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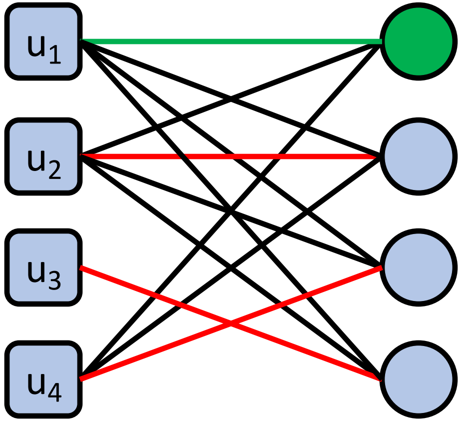
$\widehat{M}$



The `Mimic` algorithm

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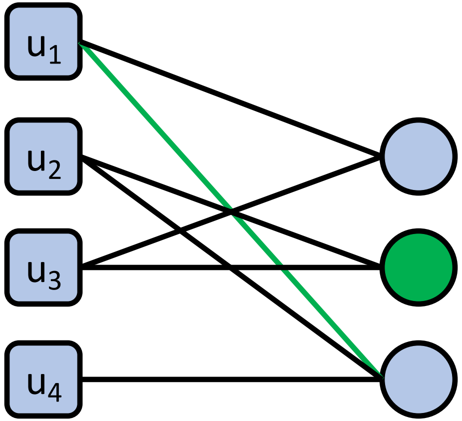

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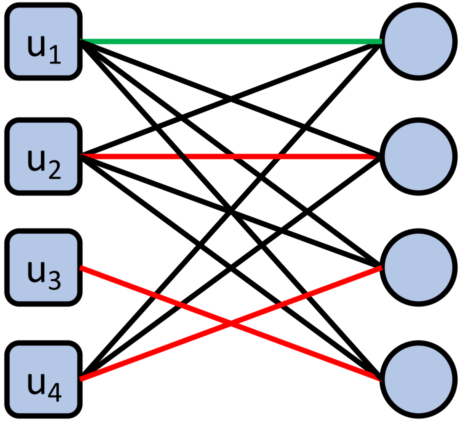

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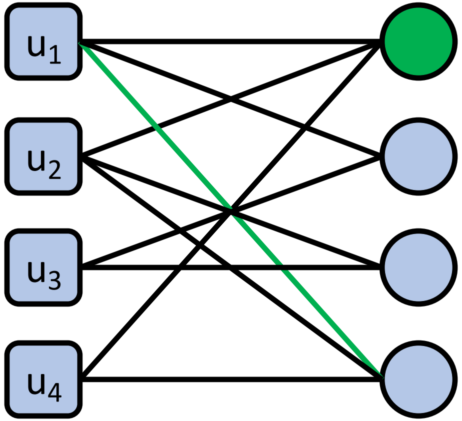

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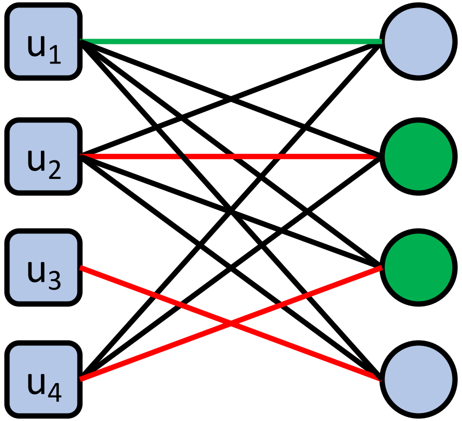

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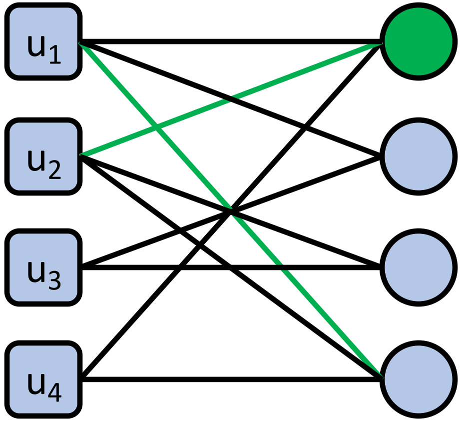

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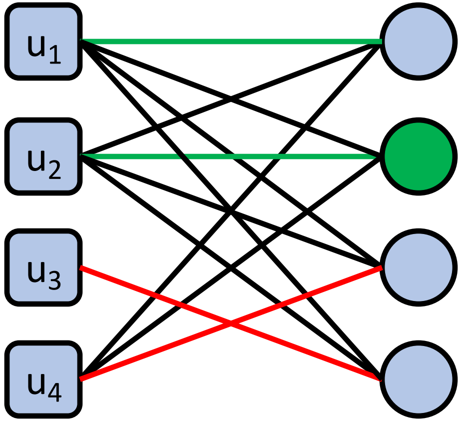

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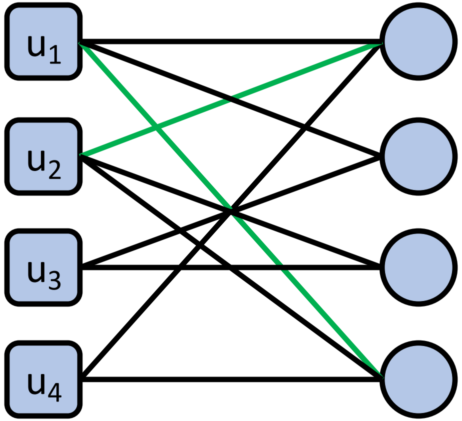

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Produced matching size

= 2

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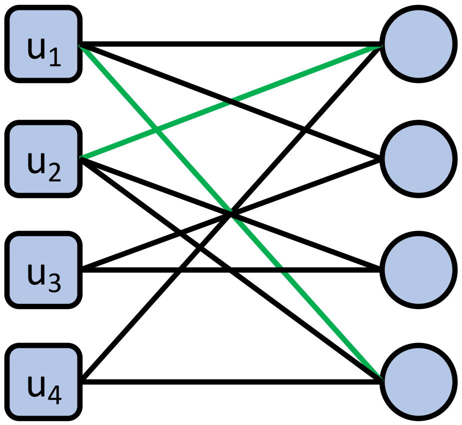

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$L_1(c^*, \hat{c})$
$= |3 - 2| + |0 - 1|$
$+|0 - 1| + |1 - 0| + 0 \ldots$
$= 4$

The Mimic algorithm

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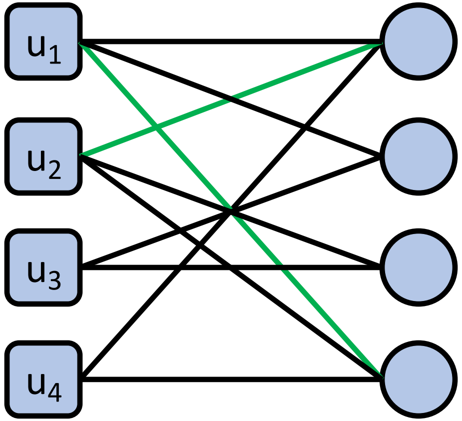

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Produced matching size

$$= 2 = \left|\widehat{M}\right| - \frac{L_1(c^*, \hat{c})}{2}$$

Error is "double counted" in L_1

$$L_1(c^*, \hat{c})$$
$$= |3-2| + |0-1|$$
$$+ |0-1| + |1-0| + 0 \ldots$$
$$= 4$$

The `Mimic` algorithm

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- Analysis
  - $0 \leq \mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}}) \leq 2\mathrm{n}$ measures how close $\hat{\mathrm{c}}$ is to c^*
  - By blindly following advice, `Mimic` gets a matching of size $|\widehat{\mathrm{M}}| - \frac{\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})}{2}$
  - `Mimic` beats an advice-free `Baseline` whenever $|\widehat{\mathrm{M}}| - \frac{\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})}{2} > \beta \cdot \mathrm{n}$

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  - By blindly following advice, `Mimic` gets a matching of size $|\widehat{M}| - \frac{L_1(c^*, \hat{c})}{2}$
  - `Mimic` beats an advice-free `Baseline` whenever $|\widehat{M}| - \frac{L_1(c^*, \hat{c})}{2} > \beta \cdot n$
  - `Mimic` beats an advice-free `Baseline` whenever $\frac{L_1(c^*, \hat{c})}{n} < 2(1 - \beta)$

For this talk, let's treat $|\widehat{M}| = n$

How to test advice quality?

Insight: Use sublinear property testing to estimate $L_1(c^*, \hat{c})$!

- Define $p = \frac{c^*}{n}$ and $q = \frac{\hat{c}}{n}$ as distributions over the 2^U types

How to test advice quality?

Insight: Use sublinear property testing to estimate $L_1(c^*, \hat{c})$!

- Define $p = \frac{c^*}{n}$ and $q = \frac{\hat{c}}{n}$ as distributions over the 2^U types
- [VV11, JHW18]: Can estimate $L_1(p, q)$ "well" using $o(n)$ IID samples
  - Some adjustments needed to apply this property testing idea to our online bipartite matching setup, but it can be done (talk to me to find out more)

[VV11] Gregory Valiant and Paul Valiant. *The power of linear estimators*. Foundations of Computer Science (FOCS), 2011.
[JHW18] Jiantao Jiao, Yanjun Han, and Tsachy Weissman. *Minimax estimation of the L_1 distance*. IEEE Transactions on Information Theory, 2018.

The TestAndMatch algorithm

- Algorithm
  - Fix any arbitrary maximum matching $\widehat{\mathrm{M}}$ on the graph defined by advice $\hat{\mathrm{c}}$
  - Run Mimic while testing quality of $\hat{\mathrm{c}}$ by estimating $\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})$
  - If test declares $\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})$ is “large”, use Baseline for remaining arrivals
  - Otherwise, continue using Mimic for remaining arrivals

The TestAndMatch algorithm

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  - If test declares $L_1(c^*, \hat{c})$ is “large”, use Baseline for remaining arrivals
  - Otherwise, continue using Mimic for remaining arrivals
- Analysis
  - If $\hat{L}_1 \lesssim 2(1-\beta)$, then TestAndMatch attains ratio of at least $1 - \frac{L_1(c^*, \hat{c})}{2n}$
  - Otherwise, TestAndMatch attains ratio of at least $\beta \cdot \left(1 - o(1)\right)$

Our second main result

Can we get an algorithm that is both 1-consistent and $\cancel{\left(1-\frac{1}{e}\right)}$ β-robust?

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and $\beta \cdot \left(1 - o(1)\right)$, depending on advice quality

- Our method is a meta-algorithm that uses any `Baseline` that achieves β
- So, we are simultaneously 1-consistent and $\beta \cdot \left(1 - o(1)\right)$-robust
- For random arrival model, we know that $0.696 \le \beta \le 0.823$

Our second main result

Can we get an algorithm that is both 1-consistent and $\cancel{\left(1-\frac{1}{e}\right)}$ β-robust?

Goal achievable in random order (Informal)

Let $\widehat{\mathrm{L}}_1$ be estimate of $\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})$ from $\mathrm{o}(\mathrm{n})$ vertex arrivals.
TestAndMatch achieves a competitive ratio of at least

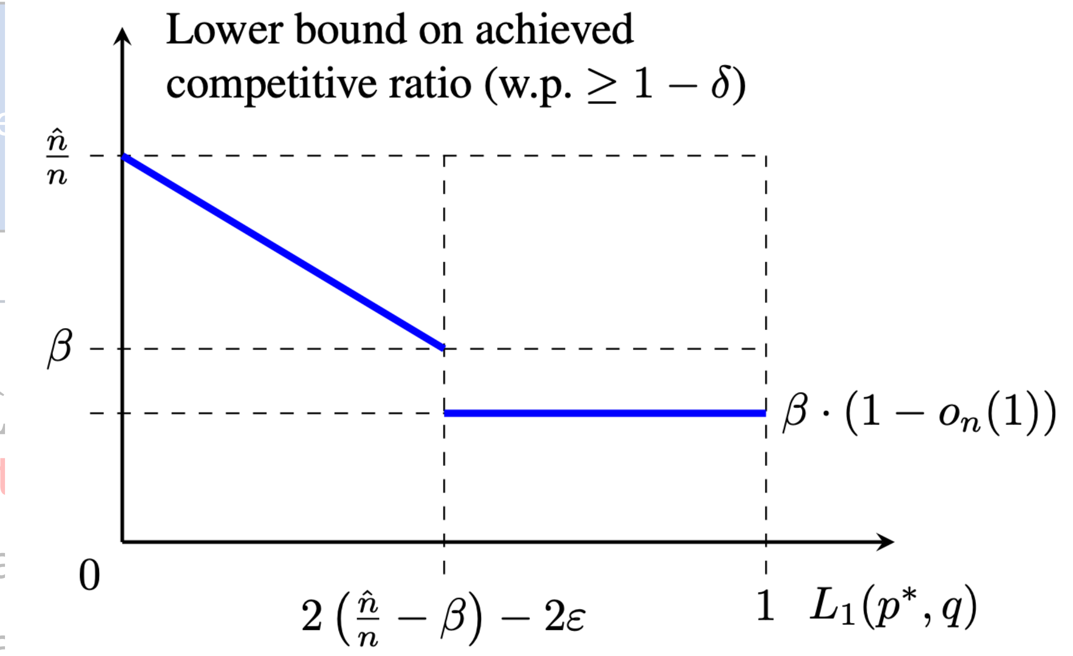
- At least $1 - \frac{\mathrm{L}_1(\mathrm{c}^*, \hat{\mathrm{c}})}{2\mathrm{n}} \geq \beta$, when $\widehat{\mathrm{L}}_1$ "small"
- At least $\beta \cdot \big(1 - \mathrm{o}(1)\big)$, when $\widehat{\mathrm{L}}_1$ "large"

i.e., TestAndMatch is 1-consistent and $\beta \cdot \big(1 - \mathrm{o}(1)\big)$-robust

Our second main result

Can we ge … obust?

β



Let $\widehat{\mathrm{L}}$ … ls.

Test … ast

- At lea…
- At lea…

i.e., TestAndMatch is 1-consistent and $\beta \cdot \big(1 - o(1)\big)$-robust

I'm on the job market!

Conclusions and future directions

- Our paper also discussed some practical considerations while using the given advice $\hat{c}$
- Can our ideas such as using property testing extend to other versions of online bipartite matching and other online problems with random arrivals?
  - We suspect it extends with suitably chosen advice and quality metrics, e.g. Earthmover distance?
- Is there a smarter way using advice other than `Mimic`, leaving some arrivals unmatched?
  - [FMMM09] constructed two matchings to "load balance" in the known IID setting
  - In semi-online model, [KPSSV19] mimic matching on known arrivals and `Ranking` on adversarial arrivals
- Message to the learning-augmented community: Beyond consistency and robustness?
  - `TestAndMatch`'s guarantees is based on L_1 over the type histograms
  - This is sensitive to certain types of noise, e.g. $\hat{c}$ obtained after Erdős–Rényi edits to the offline graph G^*
  - We expect large L_1 in practice, but notions of advice practicality are not formally considered under the standard framework of consistency and robustness

Thank you for your kind attention!

[FMMM09] Jon Feldman, Aranyak Mehta, Vahab Mirrokni, and Shan Muthukrishnan. *Online stochastic matching: Beating 1-1/e*. Foundations of Computer Science (FOCS), 2009.
[KPSSV19] Ravi Kumar, Manish Purohit, Aaron Schild, Zoya Svitkina, and Erik Vee. *Semi-Online Bipartite Matching*. Innovations in Theoretical Computer Science (ITCS), 2019.